H9: Kansverdelingen
V5 - Wiskunde C

1 a) $p(\text{hoogstens } 22) = 1 - p(\text{minstens } 23) = 1 - p(\text{som } 23) - p(\text{som } 24)$

$= 1 - \binom{4}{1}\cdot p(5666) - p(6666)$

$= 1 - 4\cdot\left(\frac{1}{6}\right)^1\cdot\left(\frac{1}{6}\right)^3 - \left(\frac{1}{6}\right)^4$

$= 1 - \frac{4}{1296} - \frac{1}{1296}$

$= \frac{1291}{1296}$

b) $p(\text{minstens } 7) = 1 - p(\text{hoogstens } 6)$

$= 1 - p(\text{som } 4) - p(\text{som } 5) - p(\text{som } 6)$

$= 1 - p(1111) - \binom{4}{1}p(1112) - \binom{4}{1}p(1113) - \binom{4}{2}p(1122)$

$= 1 - \left(\frac{1}{6}\right)^4 - 4\cdot\left(\frac{1}{6}\right)^3\left(\frac{1}{6}\right)^1 - 4\cdot\left(\frac{1}{6}\right)^3\cdot\left(\frac{1}{6}\right)^1 - 6\cdot\left(\frac{1}{6}\right)^2\left(\frac{1}{6}\right)^2$

$= 1 - \frac{1}{1296} - \frac{4}{1296} - \frac{4}{1296} - \frac{6}{1296}$

$= \frac{1281}{1296} = \frac{427}{432}$

2

1 x 100
2 x 50
4 x 10
43 x 0

3 pakken (zonder terugleggen)

a) $p(\text{minstens 1 prijs}) = 1 - p(\text{geen prijs}) = 1 - \dfrac{\binom{7}{0}\binom{43}{3}}{\binom{50}{3}} \approx 0{,}370$

b) $p(100 \text{ euro}) = p(1\times 100 \text{ en } 2\times 0) + p(2\times 50 \text{ en } 1\times 0)$

$= \dfrac{\binom{1}{1}\binom{43}{2}\binom{6}{0}}{\binom{50}{3}} + \dfrac{\binom{2}{2}\binom{43}{1}\binom{5}{0}}{\binom{50}{3}}$

$\approx 0{,}048$

c) $p(\text{minstens } 30 \text{ euro}) = 1 - p(\text{hoogstens } 20 \text{ euro})$

$= 1 - p(3\times 0) - p(1\times 10 \text{ en } 2\times 0) - p(2\times 10 \text{ en } 1\times 0)$

$= 1 - \dfrac{\binom{43}{3}\binom{7}{0}}{\binom{50}{3}} - \dfrac{\binom{4}{1}\binom{43}{2}\binom{3}{0}}{\binom{50}{3}} - \dfrac{\binom{4}{2}\binom{43}{1}\binom{3}{0}}{\binom{50}{3}}$

$\approx 0{,}173$

③ $\boxed{v}\ \square\ \square\ \square\ \square$ ← topsporters

$\frac{1}{5}$

a) $p(\text{geen enkele } v) = p(\bar{v}\bar{v}\bar{v}\bar{v}\bar{v}) = \binom{5}{0}\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^5 = \left(\frac{4}{5}\right)^5 \approx 0{,}328$

b) $p(\text{minstens } 1 \times v) = 1 - p(\text{geen enkele } v) = 1 - p(\bar{v}\bar{v}\bar{v}\bar{v}\bar{v}\bar{v}) = 1 - \binom{6}{0}\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^6$

$= 1 - \left(\frac{4}{5}\right)^6 \approx 0{,}738$

c) $p(\text{precies } 1\ v) = \binom{8}{1} p(v\bar{v}\bar{v}\bar{v}\bar{v}\bar{v}\bar{v}\bar{v}) = \binom{8}{1}\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^7 \approx 0{,}336$

$\overset{\text{of}}{=}$ binompdf$(8, \frac{1}{5}, 1) \approx 0{,}336$

④ 60% brandbaar
15% bijtend
25% anders

10 pakken (met terugleggen)

a) $p(\text{geen bijtende stoffen}) = \binom{10}{0}(0{,}15)^0(0{,}85)^{10} \overset{\text{of}}{=}$ binompdf$(10, 0.15, 0) \approx 0{,}197$

b) $p(8 \text{ brandbaar en } 2 \text{ bijtend}) = \binom{10}{8}(0{,}60)^8(0{,}15)^2 \approx 0{,}017$ (binom niet mogelijk!)

c) $p(\text{minstens } 9 \text{ brandbaar}) = \binom{10}{9}\cdot p(9 \text{ brandbaar}) + \binom{10}{10}\cdot p(10 \text{ brandbaar})$

$= \binom{10}{9}\cdot(0{,}60)^9\cdot(0{,}40)^1 + \binom{10}{10}\cdot(0{,}60)^{10}\cdot(0{,}40)^0 \approx 0{,}046$

$\overset{\text{of}}{=}$ binompdf$(10, 0.60, 9)$ + binompdf$(10, 0.60, 10) \approx 0{,}046$

⑤ a) 7: 3 R, 4 w — pakken totdat je witte pakt

x	1	2	3	4
$p(X=x)$	$\frac{4}{7}$	$\frac{2}{7}$	$\frac{4}{35}$	$\frac{1}{35}$

met X = aantal keer pakken

$p(X=1) = p(w) = \frac{4}{7}$

$p(X=2) = p(Rw) = \frac{3}{7}\cdot\frac{4}{6} = \frac{2}{7}$

$p(X=3) = p(RRw) = \frac{3}{7}\cdot\frac{2}{6}\cdot\frac{4}{5} = \frac{4}{35}$

$p(X=4) = p(RRRw) = \frac{3}{7}\cdot\frac{2}{6}\cdot\frac{1}{5}\cdot\frac{4}{4} = \frac{1}{35}$

b) optie 1-Var Stats L_1, L_2 geeft: $E(X) = 1{,}6$
$\sigma_X = 0{,}8$

⑥ U = de uitbetaling per spel
W = de winst per spel

u	3	7	11	15
$p(U=u)$	$\frac{8}{27}$	$\frac{12}{27}$	$\frac{6}{27}$	$\frac{1}{27}$

$p(U=3) = p(111) = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$

$p(U=7) = \binom{3}{1}p(511) = 3\cdot\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^2 = \frac{12}{27}$

$p(U=11) = \binom{3}{2}p(551) = 3\cdot\left(\frac{1}{3}\right)^2\cdot\left(\frac{2}{3}\right)^1 = \frac{6}{27}$

$p(U=15) = p(555) = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$

$E(U) = 3\cdot\frac{8}{27} + 7\cdot\frac{12}{27} + 11\cdot\frac{6}{27} + 15\cdot\frac{1}{27} = 7$

$E(W) = E(U) - \text{inleg} = 7 - 8 = -1$ euro

⑦ a) $p(10 < X < 15) = p(X \leq 14) - p(X \leq 10)$

$= \text{binomcdf}(25, \frac{1}{2}, 14) - \text{binomcdf}(25, \frac{1}{2}, 10) \approx 0,576$

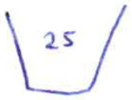
met X = aantal keer munt

b) $p(\underbrace{\text{hoogstens 10 keer}}_{X}\ \underbrace{\text{minstens 5 ogen}}_{p}) = p(X \leq 10) = \text{binomcdf}(15, \frac{1}{3}, 10) \approx 0,998$

$p = p(\text{minstens 5 ogen}) = \frac{2}{6} = \frac{1}{3}$

met X = aantal keer minstens 5 ogen

⑧



12 R
8 Z
5 W

2 pakken zonder terugleggen
15 keer experiment uitvoeren

a) $p(\underbrace{\text{3 keer}}_{X}\ \underbrace{\text{2 rode}}_{p}) = p(X = 3) = \text{binompdf}(15, 0.22, 3) \approx 0,246$

$p = p(\text{2 rode}) = \dfrac{\binom{12}{2}\binom{13}{0}}{\binom{25}{2}} = 0,22$

met X = het aantal keer 2 rode

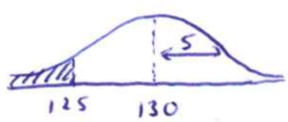
b) $p(\underbrace{\text{minstens 10 keer}}_{X}\ \underbrace{\text{precies 1 zwarte}}_{p}) = p(X \geq 10) = 1 - p(X \leq 9)$

$= 1 - \text{binomcdf}(15, \frac{34}{75}, 9) \approx 0,081$

$p = p(\text{precies 1 zwarte}) = \dfrac{\binom{8}{1}\binom{17}{1}}{\binom{25}{2}} = \dfrac{34}{75}$

met X = het aantal keer één zwarte

⑨ a)



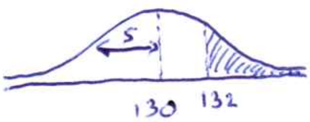
$p(\underbrace{\text{hoogstens 4}}_{X},\ \underbrace{\text{minder dan 125 gram}}_{p}) = p(X \leq 4) = \text{binomcdf}(50, 0.159, 4)$

$\approx 0,085$

$p = p(\text{minder dan 125 gram}) = \text{normalcdf}(-10^{99}, 125, 130, 5) \approx 0,159$

met X = aantal pakken met minder dan 125 gram.

b)

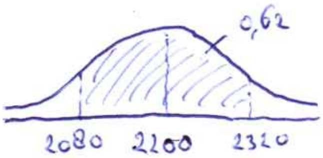


$p(\underbrace{\text{precies 8}}_{X}\ \underbrace{\text{meer dan 132 gram}}_{p}) = p(X = 8) = \text{binompdf}(50, 0.345, 8)$

$\approx 0,002$

$p = p(\text{meer dan 132 gram}) = \text{normalcdf}(132, 10^{99}, 130, 5) \approx 0,345$

⑩



Volgens de vuistregels ligt 68% binnen 1σ van μ.

Dus $\sigma_{gok} \approx 120 \rightarrow$ kies $x_{min} = 100$ en $x_{max} = 140$.

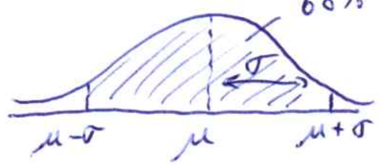
$y_{min} = 0$ en $y_{max} = 1$.

$p(2080 < X < 2320) = \underbrace{\text{normalcdf}(2080, 2320, 2200, x)}_{y_1} = \underbrace{0,62}_{y_2}$

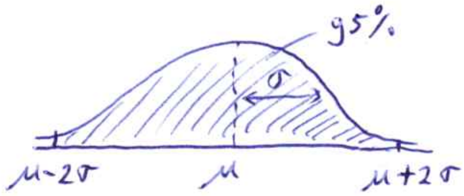
optie intersect geeft $x \approx 136,7$

Dus $\sigma \approx 140$

11 a)



b)



12

4 K
5 S
6 C

6 pakken (zonder terugleggen)

a) $p(\text{alleen colaflesjes}) = \dfrac{\binom{6}{6}\binom{9}{0}}{\binom{15}{6}} \approx 0{,}0002$ of $p(cccccc) = \frac{6}{15}\cdot\frac{5}{14}\cdot\frac{4}{13}\cdot\frac{3}{12}\cdot\frac{2}{11}\cdot\frac{1}{10}$

b) $p(\text{3 spekkies en 3 colaflesjes}) = \dfrac{\binom{5}{3}\binom{6}{3}\binom{4}{0}}{\binom{15}{6}} \approx 0{,}040$ of $\binom{6}{3}\binom{3}{3}\cdot p(sssccc) =$

$\binom{6}{3}\binom{3}{3}\cdot\frac{5}{15}\cdot\frac{4}{14}\cdot\frac{3}{13}\cdot\frac{6}{12}\cdot\frac{5}{11}\cdot\frac{4}{10}$

c) $p(\text{eerst 3 spekkies en daarna 3 colaflesjes}) = p(sssccc) = \frac{5}{15}\cdot\frac{4}{14}\cdot\frac{3}{13}\cdot\frac{6}{12}\cdot\frac{5}{11}\cdot\frac{4}{10} \approx 0{,}002$

d) $p(\text{eerst 2K en 3S en als laatste c}) = \binom{5}{2}\binom{3}{3}\cdot p(KKSSS)\cdot p(C)$

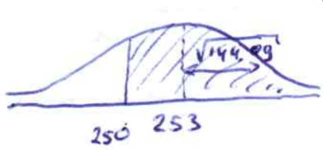
$= \binom{5}{2}\cdot\binom{3}{3}\cdot\frac{4}{15}\cdot\frac{3}{14}\cdot\frac{5}{13}\cdot\frac{4}{12}\cdot\frac{3}{11}\cdot\frac{6}{10}$

$\approx 0{,}012$

13 Het brutogewicht is $B = X + Y$

B is normaal verdeeld met $\mu_B = 5 + 248 = 253$ gram

en $\sigma_B = \sqrt{0{,}3^2 + 12^2} = \sqrt{144{,}09}$



$p(B > 250) = \text{normalcdf}(250, 10^{99}, 253, \sqrt{144{,}09}) \approx 0{,}599$

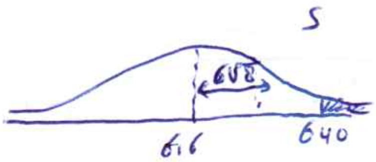
Dus in 59,9% van de gevallen is het brutogewicht meer dan 250 gram.

14 a) S = gewicht van de sinaasappels in een netje

$S = n \cdot x$

$\mu_S = n \cdot \mu_X = 8 \cdot 77 = 616$

$\sigma_S = \sqrt{n} \cdot \sigma_X = \sqrt{8} \cdot 6$



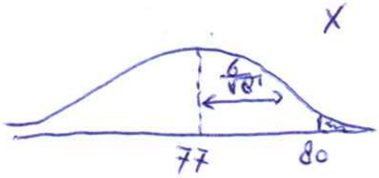
$p(S > 640) = \text{normalcdf}(640, 10^{99}, 616, 6\sqrt{8}) \approx 0{,}079$

b) $\bar{X}$ = gemiddeld gewicht van een sinaasappel uit een netje

$\bar{X} = \dfrac{S}{n}$

$\mu_{\bar{X}} = \mu_X = 77$

$\sigma_{\bar{X}} = \dfrac{\sigma_X}{\sqrt{n}} = \dfrac{6}{\sqrt{8}}$

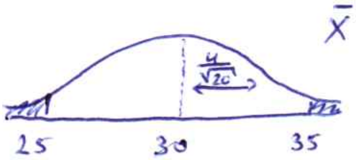


$p(\bar{X} > 80) = \text{normalcdf}(80, 10^{99}, 77, \frac{6}{\sqrt{8}}) \approx 0{,}079$

(15) a) $\mu_X = 30$ | $\mu_{\bar{X}} = \mu_X = 30$

$\sigma_X = 4$ | $\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}} = \frac{4}{\sqrt{20}}$

$n = 20$

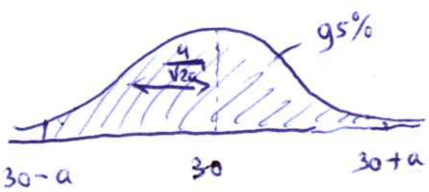


$p(\bar{X} < 25 \vee \bar{X} > 35) =$

$2 \cdot p(\bar{X} < 25) =$

$2 \cdot \text{normalcdf}(-10^{99}, 25, 30, \frac{4}{\sqrt{20}}) \approx 0{,}00000002$

b)

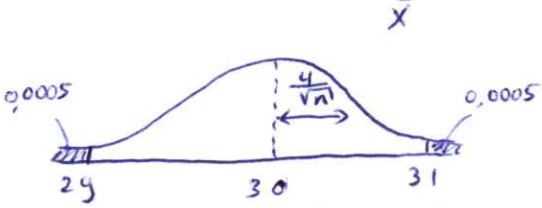


$p(30-a < \bar{X} < 30+a) = 0{,}95$

$\underbrace{\text{normalcdf}(30-x, 30+x, 30, \frac{4}{\sqrt{20}})}_{y_1} = \underbrace{0{,}95}_{y_2}$

optie intersect geeft: $x \approx 1{,}75$ | $x_{min} = 0$, $x_{max} = 2$ | $y_{min} = 0{,}9$, $y_{max} = 1$

c)



$p(\bar{X} < 29 \vee \bar{X} > 31) = 0{,}001$

$2 \cdot p(\bar{X} < 29) = 0{,}001$

$p(\bar{X} < 29) = 0{,}0005$

$\underbrace{\text{normalcdf}(-10^{99}, 29, 30, \frac{4}{\sqrt{x}})}_{y_1} = \underbrace{0{,}0005}_{y_2}$

optie intersect geeft: $x \approx 173{,}2$

Dus $n > 173$.

(16) a) $n = 1350$

$p = p(\text{succes}) = 0{,}92$

X = aantal reizen dat niet wordt geannuleerd

$p(X \leq 1250) = \text{binomcdf}(1350, 0.92, 1250) \approx 0{,}802$

b) $E(\text{aantal annuleringen}) = 1300 \cdot 0{,}08 = 104$ | $n = 1300$, $p = 0{,}08$, X = aantal annuleringen

$\sigma = \sqrt{n \cdot p(1-p)} = \sqrt{1300 \cdot 0{,}08 \cdot 0{,}92} \approx 9{,}8$

$\mu - \sigma = 104 - 9{,}8 = 94{,}2$

$\mu + \sigma = 104 + 9{,}8 = 113{,}8$

$p(94{,}2 < X < 113{,}8) = p(95 < X < 113) = p(X \leq 113) - p(X \leq 94)$

$= \text{binomcdf}(1300, 0.08, 113) - \text{binomcdf}(1300, 0.08, 94)$

$\approx 0{,}669$

(17) a) $p(\text{2 van de 3}) = \dfrac{\binom{3}{2}\binom{77}{18}}{\binom{80}{20}} \approx 0{,}1388$

b) $p(\text{3 van de 3}) = \dfrac{\binom{3}{3}\binom{77}{17}}{\binom{80}{20}} \approx 0{,}0139$

$p(\text{geen prijs}) = 1 - 0{,}1388 - 0{,}0139 = 0{,}8473$

W = uitbetaling − 1

w	−1	0	42
$p(W=w)$	0,8473	0,1388	0,0139

$\leftarrow L_1 = \{-1, 0, 42\}$

$\leftarrow L_2 = \{0{,}8.., 0{,}13.., 0{,}01..\}$

optie 1-Var Stats L_1, L_2 geeft

$E(W) \approx -0{,}26$ dollar

$\sigma_W \approx 5{,}05$ dollar

17 c) $p(2 \text{ van de } 2) = \dfrac{\binom{2}{2}\binom{78}{18}}{\binom{80}{20}} \approx 0{,}060$

$p(\text{geen prijs}) \approx 1 - 0{,}060 = 0{,}940$

w	-1	11
$p(W=w)$	$0{,}940$	$0{,}060$

$E(W) = -1 \cdot 0{,}940 + 11 \cdot 0{,}060 \approx -0{,}28$ dollar

18 a) $p(\underbrace{\text{meer dan 4 bollen}}_{X} \text{ met } \underbrace{\text{een vetpercentage van meer dan } 8\%}_{p})$

$p(X > 4) = 1 - p(X \leq 4) = 1 - \text{binomcdf}(10, 0{,}375..., 4) \approx 0{,}306$

met X = aantal bollen met een vetpercentage van meer dan 8 %

en $p = \text{normalcdf}(8, 10^{99}, 7.3, 2.2) \approx 0{,}375..$

b) $\underbrace{\text{normalcdf}(8, 10^{99}, 6.9, \sigma)}_{y_1} = \underbrace{\frac{12}{50}}_{y_1}$

optie intersect geeft: $\sigma \approx 1{,}6\ \%$

19 S = dikte van 20 tabletten

$\mu_S = n \cdot \mu_X = 20 \cdot 0{,}81$

$\sigma_S = \sqrt{n} \cdot \sigma_X = \sqrt{20} \cdot 0{,}05$

L = binnenlengte van een buisje

De tabletten passen niet in een buisje als $S > L$, dus $S - L > 0$

Stel $V = S - L$

$\mu_V = \mu_S - \mu_L = 16{,}20 - 17{,}50 = -1{,}30$ cm

$\sigma_V = \sqrt{(0{,}05\sqrt{20})^2 + (0{,}48)^2} \approx \sqrt{0{,}2804}$ cm

$p(V > 0) = \text{normalcdf}(0, 10^{99}, -1{,}30, \sqrt{0{,}2804}) \approx 0{,}007.$